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Application Note 0902

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Analytical Model for C1MJ and C3MJ CDO-100 Solar Cells and CCAs

Introduction

The model presented below is made available to customers who want to model the behavior of Spectrolab CPV solar cells in their systems. The model provides a functional representation of the I-V characteristics and light response of the cell using the standard Shockley equation and an empirical fit to the data. For CCAs a standard Shottky diode model with empirically determined values is presented.

Nomenclature and Empirical Model Values

Symbol	*Definition*	*C1MJ Value*	*C3MJ Value*
I	Current (Amps)	—	—
V	Voltage (Volts)	—	—
X	Concentration (suns)	—	—
I_{SAT}	Reverse saturation current (Amps)	$1(10^{-20})$ at 25°C	$1(10^{-20})$ at 25°C
J_{SAT}	Reverse saturation current density (Amps/cm^2)	$1.01(10^{-20})$ at 25°C	$1.01(10^{-20})$ at 25°C
A_A	Aperture area (cm^2)	0.989 cm^2	0.989 cm^2
N	Ideality factor (no units)	2.44	2.57
Q	Electron charge (1.6×10^{-19} Coulombs = 1 eV)	—	—
K	Boltzmann constant (8.62×10^{-5} eV/°K)		
$R_{S\infty}$	Series resistance at high flux (Ohms)	11 mΩ	13.6 mΩ
R_{S0}	Series resistance at low intensity (Ohms)	40 Ω	40 Ω
κ	Series resistance intensity coefficient	1.75	1.75
T	Temperature (°K)	—	—
W	Incident solar radiant intensity (Watts/cm^2)	0.0901 W/cm^2	0.0901 W/cm^2
$\mathcal{R}$	Responsivity (Amps/Watt)	0.139 Ampss/Watt	0.1405 Amps/Watt
E_g	Effective energy gap (eV)	1.6 eV	1.6 eV
Γ	I_{SAT} temperature parameter (no units)	1	1
I_{SC}	Short-circuit current (Amps)	calculated	Calculated
V_{OC}	Open-circuit voltage (Volts)	calculated	Calculated
I_{MP}	Current at maximum power (Amps)	calculated	Calculated
V_{MP}	Voltage at maximum power (Volts)	calculated	Calculated
FF	Fill factor (no units)	calculated	Calculated
H	Efficiency (no units)	calculated	Calculated
Symbol	***Definition***	***Bypass Diode Value***	
A	Richardson constant (120 Amps/cm^2 °K^2)	120 Amps/cm^2 °K^2	
A^*/A	Effective Richardson constant for p-type Si[3]	0.66	
A_B	Bypass diode area (cm^2)	0.1925 cm^2	
φ_{Bn}	Effective barrier height (eV)	0.65 eV	
n_B	Bypass diode ideality factor	1.2	
R_{SB}	Bypass diode series resistance (Ohms)	12 mΩ	
R_{SHB}	Bypass diode shunt resistance (Ohms)	1.6×10^5 Ω	

The Cell Model

The current-voltage characteristics of a solar cell are given by [1]

$$I = -I_{SAT}\left[\exp\left(\frac{q(V + IR_S)}{nkT}\right) - 1\right] + I_L$$

where $I_{SAT} = J_{SAT}A_A$ and $I_L = J_L A_A$. J_L is given by $J_L = \mathcal{R}W$. The signs of the currents are reversed from the usual diode equation because by convention in the solar industry, the power producing quadrant is taken as positive voltage and positive current. For R_S other than zero, this equation must be solved iteratively for I. Figure 1 shows the modeled I-V characteristic at 555 suns (50 W/cm^2 radiant intensity) and 25°C for a CDO-100 C3MJ “nominal” cell.

It is found that to obtain an adequate fit to the data, the series resistance is dependent on the incident power, and is modeled as

$$R_S = R_{S0}/X^{\kappa} + R_{S\infty}$$

This function is plotted in Figure 2. There are some plausible explanations of this behavior in which the effective resistance is a function of photo-induced injection levels but, as with all the other parameters of the model herein, the equation for R_S is simply one that empirically matches the data sheet measurements reasonably well.

Temperature dependence is due to the temperature term in the above equation and by the temperature dependence of the saturation current [2],

$$J_{SAT}(T) \sim T^{3+\gamma/2}\exp\left(-\frac{E_g}{kT}\right),$$

so given a value of J_{SAT} at a reference temperature, its value at other temperatures can be calculated from this proportionality.

With the cell I-V characteristics expressed in this form, the key performance parameters can be readily calculated. The short circuit current I_{SC} is just I_L for reasonable values of R_S, and the open circuit voltage is

$$V_{OC} = \frac{kT}{q}\ln\left(\frac{I_L}{I_{SAT}} + 1\right).$$

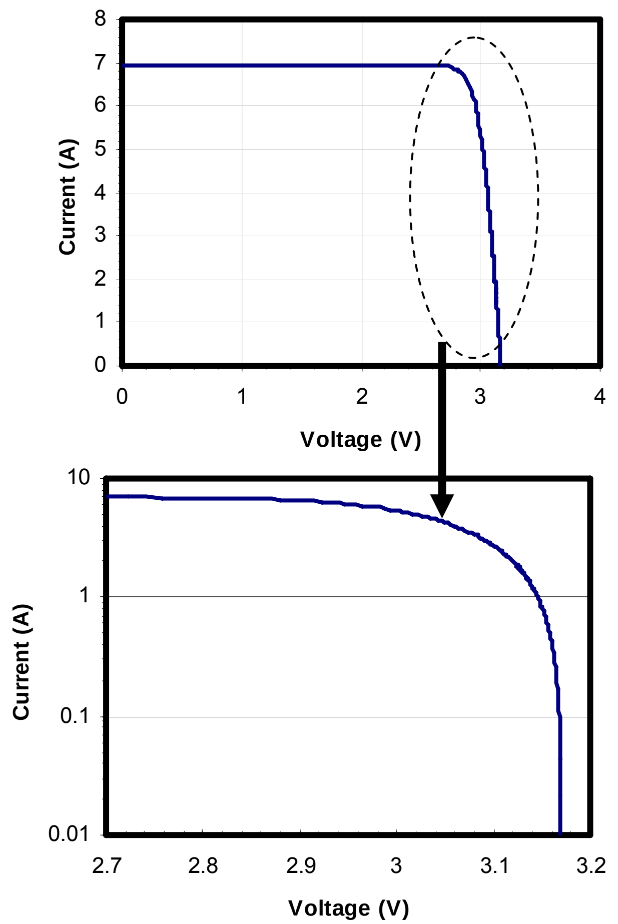

Figure 1. Modeled I-V characteristic of a C3MJ CDO-100 nominal cell at 25°C and 555x concentration (50 W/cm²).

The current and voltage at maximum power are given by

$$\ln\left(\frac{I_L - I_{MP} + I_{SAT}}{I_{SAT}}\right) - \frac{I_{MP}}{I_L - I_{MP} + I_{SAT}} - \frac{2R_S I_{MP}}{nkT} = 0$$

and

$$V_{MP} = \frac{nkT}{q}\ln\left(\frac{I_L}{I_{SAT}} - \frac{I_{MP}}{I_{SAT}} + 1\right) - I_{MP}R_S\,.$$

Here we again observe the convention that current is positive in the power producing region, hence the signs of terms involving I_{MP} have been adjusted from those shown in ref [1]. The solution for I_{MP} must be found iteratively.

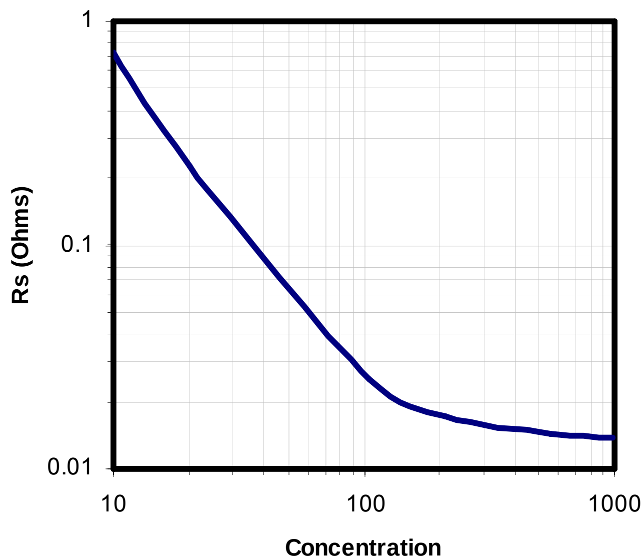

Figure 2. Empirically fitted series resistance as a function of concentration.

Fill factor is given by

$$FF = \frac{I_{MP}V_{MP}}{I_{SC}V_{OC}}$$

and efficiency is

$$\eta = \frac{I_{MP}V_{MP}}{WA_A}\,.$$

Figure 3 shows plotted data for V_{MP} and η along with the modeled curves for the C3MJ cell. It can be seen that the model fits reasonably well but not perfectly. In particular, it can be seen that the model predicts higher V_{MP} at 1000 suns concentration, suggesting that R_S is understated; however, the value used for R_S does result in good agreement for the efficiency versus concentration curves.

The CCA Model

A CCA using either C1MJ or C3MJ cells can be modeled using the solar cell model above in combination with a model for the Shottky bypass diode. The standard model for a Shottky diode is based on thermionic emission through the metal-semiconductor barrier[3]; we use this standard model with series and shunt resistance terms:

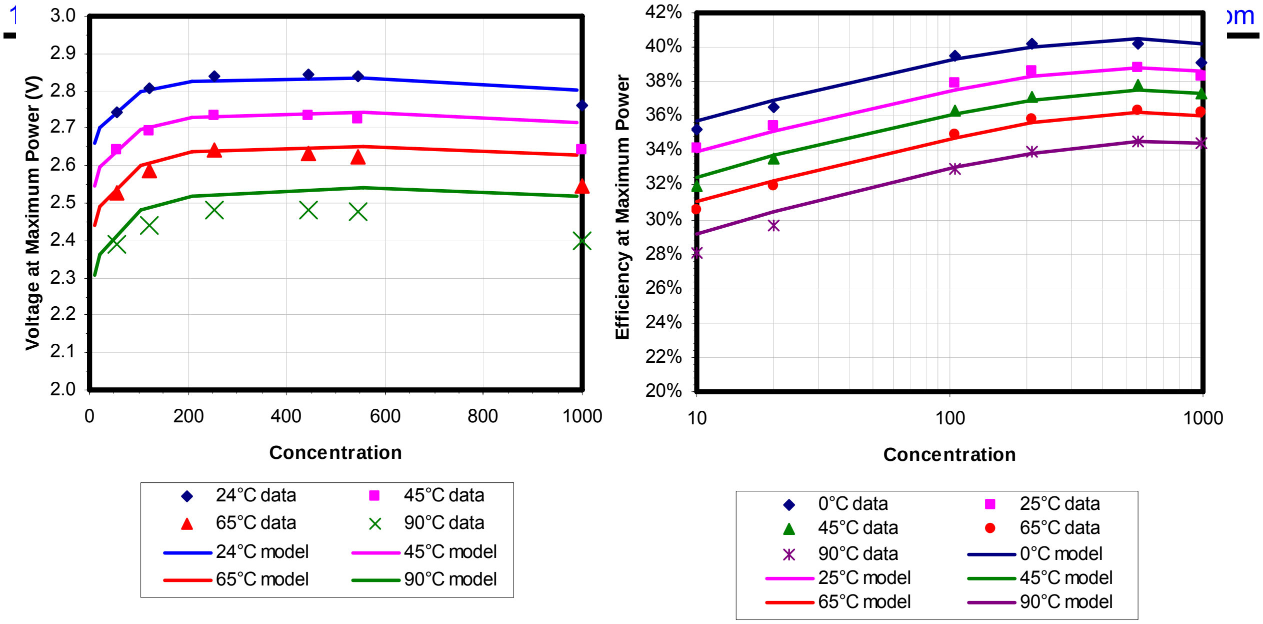

Figure 3. Modeled temperature dependence of V_{MP} and η as a function of concentration.

$$I = I_{ST}\exp\left[\frac{q(V - IR_{SB})}{nkT} - 1\right] + \frac{V}{R_{SHB}}$$

where

$$I_{ST} = A_B A^* T^2 \exp\left(\frac{-q\phi_{Bn}}{kT}\right).$$

A*, the effective Richardson constant, is 0.66×120 = 79.2 Amps/cm^2°K^2.

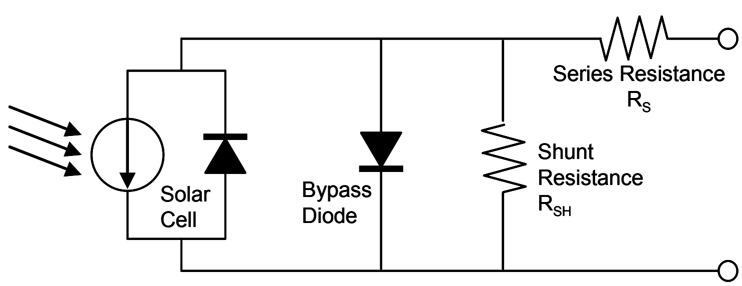

Figure 4. CCA Equivalent Circuit

An equivalent circuit for the CCA is shown in Figure 4. However, with the parallel-gap welded silver interconnects used in Spectrolab CCAs, the added series resistance of the CCA is negligible, and the shunt resistance of the ceramic substrate is similarly negligible. Thus the I-V characteristic of the CCA is obtained by simply summing the currents from the previously described solar cell and bypass diode models (reversing the polarity of the bypass diode as reflected in the equivalent circuit). Figure 5 plots the forward and reverse I-V characteristics of the bypass diode. The resulting I-V characteristics for a CCA with the C1MJ cell are shown in Figure 6.

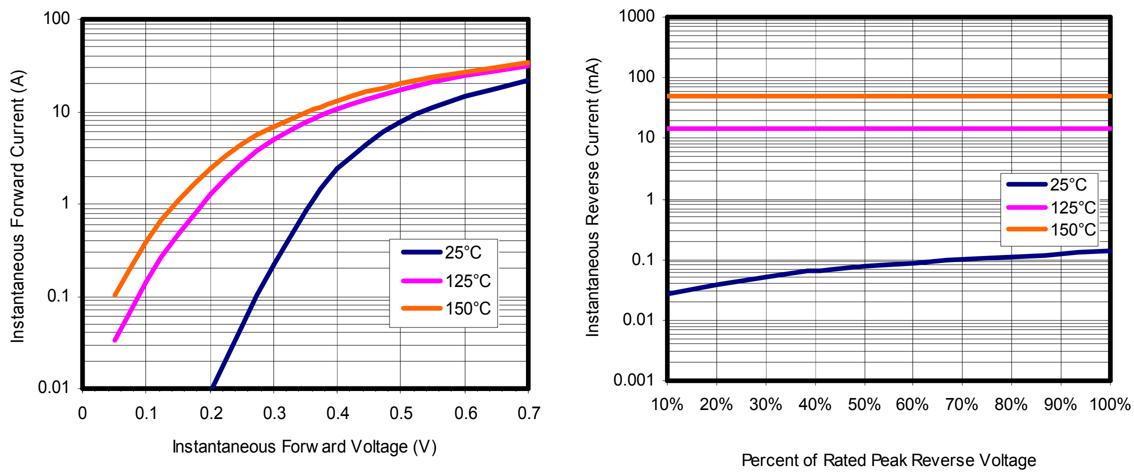

Figure 5. Modeled temperature dependence of Shottky bypass diode.

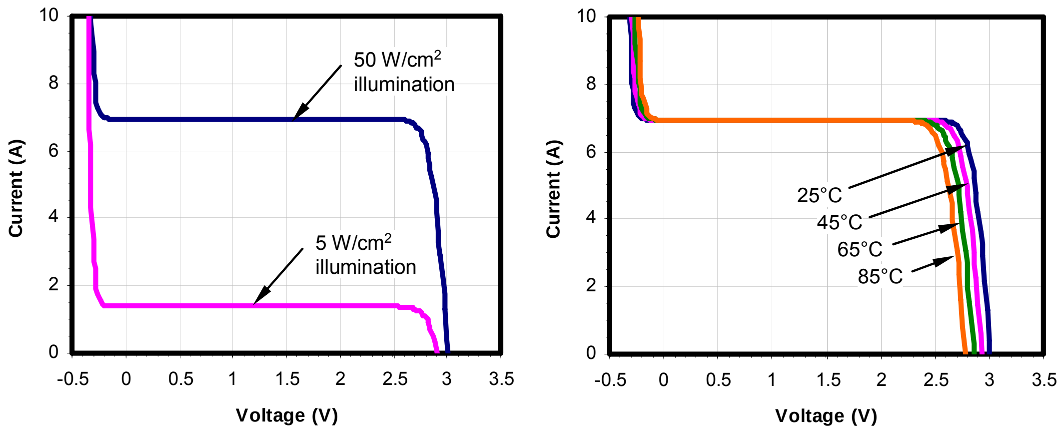

Figure 6. Modeled CCA I-V Characteristics (C1MJ cell).

Remarks

It should be emphasized that the forgoing is an adaptation of the conventional textbook model of a solar cell to real-world triple-junction cells and as such, provides a convenient means for modeling the behavior of the cell at a fairly high level. On the other hand, the model parameters used are strictly an empirical fit with no real correspondence to the actual semiconductor junctions involved. As such, the model should be used with caution. A more rigorous treatment would model each of the sub-cells as individual solar cells, each with its own I-V characteristic, and connected in series, but such detailed modeling is beyond the scope of this work.

References

[1] Partain, *Solar Cells and their Applications*, Chapter 1

[2] Sze, *Physics of Semiconductor Devices*, 2nd Edition, p. 88, eq. 46.

[3] Sze, *Physics of Semiconductor Devices*, 2nd Edition, p. 256–257 and p. 274, Fig. 16.